ECON 3510 - Intermediate Macroeconomic Theory

Fall 2015

Mankiw, *Macroeconomics, 8th ed.*, Chapter 8

Chapter 8: Economic Growth: Capital Accumulation and Population Growth

Key points:

- Solow Growth Model
- Know how to solve for the steady state of an economy
- Know how to find the Golden Rule capital stock
- Solow Growth Model with population growth

Intro

- Robert Lucas: "Once you start thinking about economic growth, it's hard to think about anything else."
- Economic growth is the central question of economics - how to make people better off.
- We'll focus on our models of consumption and output - but income and well-being are highly correlated.
- Consider these countries: Niger, India, US
- US steady growth high income
- India has been closing the gap with US, India gained independence in 1947 from England
- Niger, gained independence from France in 1960 - stagnant since for most part
- Why diff from india?
- Look at diff in outcomes...

The per-capita production function:

- Recall the aggregate production function: $Y = F(K, L)$
- Assume CRS $\Rightarrow zY = F(zK, zL)$
- Let $z = \frac{1}{L}$ (i.e., divide everything by the number of workers)
  - CRS $\Rightarrow F(\frac{1}{L}K, \frac{1}{L}L) = \frac{1}{L}Y$
  - $\Rightarrow \underbrace{\frac{Y}{L}}_{\text{output per worker}} = F(\underbrace{\frac{K}{L}}_{\text{capital per worker}}, 1)$
- New notation:
  - $\frac{Y}{L} = y$
  - $\frac{K}{L} = k$

  - $F(k, 1) = f(k)$
  - $\Rightarrow$ per worker production function:
    * $y = f(k)$
    * $MPK = \frac{\partial f(k)}{\partial k}$ or $MPK = f(k+1) - f(k)$
    * $\rightarrow$ will assume diminishing marginal product (i.e., $\frac{\partial MPK}{\partial k} < 0$)
    * DRAW concave production function

<u>The demand for goods</u>:

- Demand for C & I
  - Assume closed economy
  - Ignore gov't (both of these are temporary assumptions)
- Put in per worker terms:
  - $c = \frac{C}{L}$
  - $i = \frac{I}{L}$
- $\Rightarrow y = c + i$
  - Output per worker split between consumption and investment
- Assume save some fraction, s, of income
  - $\Rightarrow$ consumption function $= c = (1-s)y$
  - $0 \leq s \leq 1$
- Put consumption function in national act's identity:
  - Assume G, T, $NX = 0$
  - $\Rightarrow y = c + i = (1-s)y + i$
  - $\Rightarrow y = (1-s)y + i$
  - $\Rightarrow \underbrace{i}_{\text{invest}} = \underbrace{sy}_{\text{savings}}$
- DRAW production function, savings function, and note about of consumption and investment per worker

<u>Capital Accumulation</u>:

- $\rightarrow \underbrace{k_{t+1}}_{\text{capital tomorrow}} = \underbrace{k_t}_{\text{capital today}} - \underbrace{\delta k_t}_{\text{deprec capital}} + \underbrace{i_t}_{\text{investment}}$
- DRAW deprecation as a function of k (straight line with slope $= \delta$)
- Capital accumulation equation: $k_{t+1} = (1-\delta)k_t + i_t$

<u>Solow Growth Model</u>:

- A model that shows how growth in capital stock, population, and technology affect total output

- 3 main pieces:
  1. Per worker production function: $y = f(k)$
  2. Savings/invest behavior: $i = sy$
  3. Capital accumulation equation: $k_{t+1} = (1-\delta)k_t + i_t$
- Steady state
  - Since the model will have an infinite horizon, we will consider the steady state, which will represent what happens in the long run
  - The steady state is defined as the point where the growth rates of the economy and the factors of production are constant (maybe 0)
  - The capital stock will grow only up to some point:
    * as $k \uparrow$, $\delta k \uparrow$
    * $\Delta k = k_{t+1} - k_t = (1-\delta)k_t + i_t - k_t$
    * $\Rightarrow \Delta k = i_t - \delta k_t$
    * $\Rightarrow \Delta k = sf(k_t) - \delta k_t$
    * b/c of diminishing MPK, at some point:
    * $\rightarrow \Delta k = sf(k_t) - \delta k_t = 0$
    * $\rightarrow$ i.e., investment = depreciation: $sf(k_t) = \delta k_t$
  - The k_t that makes the above hold is call the steady state capital stock, we done it by k^*
  - This is the long-run equilibrium of the economy
  - DRAW the production function, savings function, depreciation function together and note the SS capital stock
  - Converge to k^* from both above and below the SS amount
- <u>Example: Working through the Solow Model</u>:
  - need to know: $s, \delta, f(k)$ and initial capital k_1
  - Let $f(k) = k^\alpha$, $\alpha = \frac{1}{3}$, $\delta = 0.10$, $s = 0.2$, $k_1 = 1$

Table 1: Add caption

year	k	y	i	c
1	1.000	1.000	0.200	0.800
2	1.100	1.032	0.206	0.826
3	1.196	1.061	0.212	0.849
4	1.289	1.087	0.217	0.870
5	1.378	1.111	0.222	0.889
6	...			

  - $k_1 = 1$
  - $\Rightarrow y_1 = f(k_1) = k_1^\alpha = 1^{1/3} = 1$
  - $\Rightarrow i_1 = sy_1 = 0.2 * 1 = 0.2$
  - $\Rightarrow c = y - i = 1 - 0.2 = 0.8$
  - $\Rightarrow k_2 = (1-\delta)k_1 + i_1$
    * $= (1 - 0.1)1 + i_1$
    * $= (0.9 * 1) + 0.2 = 1.1$

How to find the Steady State:

- At steady state: $\Delta k = \underbrace{sf(k)}_{i} - \delta k = 0$
  - $\Rightarrow sf(k) = \delta k$
  - $\Rightarrow \frac{k}{f(k)} = \frac{s}{\delta}$
  - $\Rightarrow k^*$ solves: $\frac{k^*}{f(k^*)} = \frac{s}{\delta}$
  - w/ numbers from our example:
    * $\frac{k^*}{k^{*1/3}} = \frac{s}{\delta} = \frac{0.2}{0.1} = 2$
    * $\Rightarrow k^{*2/3} = 2$
    * $k^* = 2^{3/2} = 2.82$
  - SHOW excel spreadsheet to calc....
- $\uparrow$ savings rate $\Rightarrow \uparrow k^*$
- DRAW savings functions with deprecation function. Show how higher savings rate crosses deprecation function at higher value so higher SS capital stocks
- SHOW graphs of savings vs investment
- NOTE that in SS, there is no growth in K or k

The Golden Rule Level of Capital:

- Gov't policies (e.g., taxes on capital) can affect savings behavior $\Rightarrow$ affect steady state capital stock
- If one could choose "s", what would be the best savings rate? i.e. what is the best steady state?
  - Assumption on preferences: people like to consume $\Rightarrow$ want SS with highest c
  - $y = i + c$
  - $\Rightarrow c = y - i$
  - $\Rightarrow c = f(k) - \underbrace{sf(k)}_{=\delta k \text{ in SS}}$
  - $\Rightarrow c^* = f(k^*) - \delta k^*$
  - want to max c^*? Remember calculus..,
  - $\frac{\partial c^*}{\partial k^*} = \underbrace{\frac{\partial f(k^*)}{\partial k^*} - \delta = 0}_{\text{holds at max}}$
    * $\Rightarrow MPK^* - \delta = 0$
    * $\underbrace{\Rightarrow MPK^* = \delta}_{k^*_{gold} \text{ solves this}}$
    * use eq'n for SS to get the s that results in this SS cap stock:
    * $sf(k^*_{gold}) = \delta k^*_{gold}$ - the s that makes this eq'n hold is the optimal savings rate - the rate that returns k^*_{gold}.
    * DRAW graph with production function, savings function, depreciation function and show how find c* gold (largest distance between deprecation function and output function)

    - What the calculus means is that you maximize consumption by investing in capital up until the point the the additional output from that capital just offsets additional depreciation of that capital.
  - Example: $\delta = 0.1$, $f(k) = k^{\alpha}$, $\alpha = \frac{1}{3}$
    - This means that the $MPK = \alpha k^{\alpha-1} = \frac{1}{3}k^{\frac{-2}{3}}$
    - Thus, k^*_{gold} solves: $\frac{1}{3}k^{\frac{-2}{3}} = 0.1$
    - $\Rightarrow k^*_{gold} = (0.3)^{\frac{-3}{2}} = 6.086$
    - What is the savings rate needed to get here?
    - Solve for s: $s(f(k^*_{gold})) = \delta k^*_{gold}$
    - $\Rightarrow s = \frac{\delta k^*_{gold}}{f(k^*_{gold})} = \frac{\delta k^*_{gold}}{(k^*_{gold})^{\alpha}} = \delta (k^*_{gold})^{1-\alpha}$
    - $s = 0.1(6.086)^{\frac{2}{3}}$
    - $\Rightarrow s = \frac{1}{3}$

Transition to the Golden Rule Capital Stock:

- Start w/ $k^* > k^*_{gold}$
  - $\Rightarrow$ Saving too much
  - $\Rightarrow$ must reduce savings rate to reach k^*_{gold}
    - $i\downarrow, c\uparrow, y\downarrow$ immediately
    - $i\downarrow, c\uparrow, y\downarrow$ in long run
  - DRAW impulse response functions showing that cons increase and invest decrease immediately, but then cons converges back towards ss???
  - NOTE that get more consumption immediately and in long run
- Start w/ $k^* < k^*_{gold}$
  - $\Rightarrow$ Saving too little
  - $\Rightarrow$ must increase savings rate to reach k^*_{gold}
    - $i\uparrow, c\downarrow, y\uparrow$ immediately
    - $i\uparrow, c\uparrow, y\uparrow$ in long run
  - DRAW impulse response functions showing that cons decrease and invest increase immediately, can increase to higher level in long run
  - NOTE that get less consumption immediately BUT more in long run

Population Growth:

- Now L not fixed, but growing at rate n
  - $\Rightarrow \Delta k = i - (\underbrace{\delta}_{\text{deprec capital stock}} + \underbrace{n}_{\text{less cap stock per person as pop grows}})k$
  - $\Rightarrow \Delta k = sf(k) - (\delta + n)k$
    - DRAW savings function with "total deprecation" $(\delta + n)$ function. Show k*
    - "break-even" where investment offsets depreciation and pop growth (i.e., $\Delta k = 0$)
- Effects of population growth on k^*

- DRAW savings function with "total deprecation" $(\delta + n)$ function. Show k* for two value of n...
- $\Rightarrow \frac{\partial k^*}{\partial n} < 0$
  * SS capital stock per person lower with higher population growth
  * Note that in SS, K grows at a rate of n, but there is no growth in k

- Effects of population growth on k^*_{gold}
  - Still max c^*
  - $c = y - i$
    * $\Rightarrow c = f(k^*) - (\delta + n)k^*$ in SS
    * $\Rightarrow \frac{\partial c}{\partial k^*} = \frac{\partial f(k^*)}{\partial k^*} - \delta - n = 0$
    * $\rightarrow\Rightarrow MPK(k^*) = \delta + n$
    * What this calculus means is that you want to invest in capital up to the point the the additional output per unit of capital just offsets the additional depreciation and the rate of population growth (which implies how that additional capital is spread over people).
    * Alternatively, we can write this as: $MPK - \delta = n$.
    * That is, invest in capital until the additional output, net of deprecation, just equals the population growth rate. If higher - invest more and MPK falls. If lower, invest less, and MPK rises.
    * $\rightarrow\Rightarrow MPK(k^*)$ higher with $n > 0$
    * $\rightarrow\Rightarrow k^*_{gold}$ with pop growth $< k^*_{gold}$ w/o population growth
    * $\rightarrow$ i.e., $\frac{\partial k^*_{gold}}{\partial n} < 0$

<u>Views on Population Growth:</u>

- Malthusian
  - Some say "Dismal Science" comes from Thomas Malthus' view of pop growth
    * It apparently comes from economists support for abolition in the 19th century
  - Idea: population growth exhausts resources
  - Hypothesis: Income per capita declines as population grows
  - SHOW figure 8-xx (income levels and pop growth)
- Kremerian
  - Idea: Pop growth increases the most important resources of all: ideas, markets
  - Hypothesis: Income per capita increases as population grows
- Want to bet?
  - Paul Ehrlich (biologist and author of *Population Bomb*) made a public wager with economist Julian Simon
  - The Ehrlich-Simon wager was a bet that the price of 5 metals would not increase in 10 years (this was Simons side of the bet), 1980-1990
  - Ehrlich choose the metals: Copper, chromium, nickel, tin, and tungsten
  - Simon won the bet